On Growth of Limited Scale-Free Overlay Network Topologies

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***Abstract*—Overlay network topologies provide different networking applications an abstraction over underlying network architecture. Therefore, their construction and the resulting topological characteristics play a crucial role in the performance of operations running in these applications. Thanks to their small diameters, scale-free (power-law) overlay network topologies are one of the structures that offer high performance for these networks. However, a key problem for such networks is the high connectivity (i.e., load) in only a small portion (i.e., hubs) of nodes. In fact, the nodes in such scale-free overlay networks may not want or be able to accomplish such high connectivity due to technical restrictions. Therefore, some hard cutoffs are often imposed on the number of edges that each node can have, making them limited scale-free networks. In this paper, we discuss and analyze the growth of such limited scale-free networks and propose an algorithm aiming to achieve perfect scale-free overlay network topologies with low communication overhead and without global information usage during its construction phase. Through extensive simulations, we also evaluate the proposed approach and show its superiority over the existing solutions.**

I. Introduction

Overlay networks have recently gained popularity since they offer enhanced functionality to end-users by forming an independent virtual network over the underlying native (i.e. physical) layer. Today many applications (e.g. peer-to-peer networks) use overlay networks for abstraction from the underlying layers.

The topological characteristics of the formed overlay networks have profound impact on the efficiency of operations in the corresponding applications (e.g. searching in peer-to-peer networks). Clearly, if it is applicable in real deployment, one of the best performances can be achieved when the overlay topology is scale-free or has power-law degree distribution thanks to small network diameter of such topologies. However, constructing such scale-free overlay topologies is challenging. Also, in these networks, a very few percent of nodes (i.e. hubs) are required to have high connectivity (i.e. load). However, the nodes/peers may not want or be able to accomplish such high connectivity due to technical restrictions. Therefore, some hard cutoffs are often imposed on the number of edges that each node can have, making them *limited scale-free networks* [16]. Obviously, as the value of hard cutoff decreases, the diameter of the network increases, thus the worst case performance of the operations of applications may decrease.

In this paper, we focus on the construction of scale-free overlay network topologies which combine the benefits of networks with scale-free property and overlay network abstraction. Such networks have been utilized in several applications including peer-to-peer networks [1], wireless ad hoc and sensor networks [2] and even database networks [3] thanks to various performance improvements that they offer. Having the aforementioned challenges and limitations, we propose a growth model that constructs a scale-free overlay network topology having the following properties: (i) *high adhered to scale-freeness:* as the more the overlay networks adhere to scale-free property, the more benefit the applications get from scale-free features, (ii) *limited:* so the practical limits on node connectivity can be imposed, (iii) *cost-efficient:* as it reduces the communication overhead incurred during construction and does not use any global topology information, and (iv) *parameterized:* as the desired post-construction parameters (i.e. γ) of the scale-free network can be predetermined.

The rest of the paper is organized as follows. In Section II, we give a background on scale-free overlay networks and talk about previous work on growth models for scale-free networks. In Section III, we present our analysis on the growth of limited scale-free networks and in Section IV, we continue with the details of the proposed growth model. Section V presents the simulation results in which we evaluate the performance of the proposed model and compare it with existing work in terms of several metrics. Finally, we end up with conclusion in Section VI.

II. Background

A. Scale-free overlay networks

Scale-free networks have attracted a great deal of research interest since the discovery of scale-free property in many natural and artificial systems such as the Internet [4] and scientific collaboration network [5]. In these networks, nodes are connected according to the power-law degree distribution. That is, the degree distribution of nodes does not depend on

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the number of nodes in the network. The probability that a node has degree i is proportional to $P(i) \approx i^{-\gamma}$, where the exponent is often limited to the range $2 \leq \gamma \leq 3$. However, in limited scale-free networks, only nodes with degrees smaller than the hard cutoff comply with this rule. We will elaborate on this later.

Scale-free networks have many interesting properties such as high tolerance to random failures and attacks [6], high synchronizability [7], and resistance to congestion [8]. Moreover, scale-free networks also have small-world properties. Their diameters or the mean hop distance between their nodes scales with the system size logarithmically, in the range of $[O(\ln(\ln(n))), O(\ln(n))]$, depending on network parameters [9][1]. To benefit from these properties, scale-free overlay network topologies have been used in several applications. In peer-to-peer networks [10], they have been used to make these networks scalable and increase the search efficiency. In wireless ad hoc and sensor networks [2], high degree overlay nodes are placed at the physical nodes with more power and often serve as the network's routers. Moreover, in database networks (e.g. GaianDB [3]), they have been used to reduce the cost of query retrieval from database nodes.

B. Growth of Scale-Free Networks

There have been many growth models proposed for scale-free networks in network science. Barabasi-Albert (BA) model [12] is one of the well-known algorithms in the literature proposed to generate scale-free networks. It applies 'preferential attachment (PA)[2]' of new joining nodes to existing nodes in the network. Each joining node selects to connect to existing node j with probability $p_c(j)$ that is proportional to the existing node's current degree, d_j. Thus, with n denoting the total node count in the network:

$$p_c(j) = \frac{d_j}{\sum_{i=1}^{n} d_i}$$

Each joining node computes $p(j)$ for each existing node in the network and selects k (predefined growth model parameter) of them to connect to. The network formed by BA model simply produces a power-law degree distribution with $\gamma = 3$, thus $P(i) \approx i^{-3}$. There are also other models that differ from the BA model in terms of the function used to compute $p_c(j)$. However, they all adopt the idea of preferential attachment. Interested readers can review different models in [15].

One of the important characteristics of scale-free networks is the natural cutoff of the node degrees due to the finite network size effect. Natural cutoff as defined in [15] is the value of the degree such that at most one vertex can be found with the higher degree. It is computed as $\approx kn^{1/(\gamma-1)}$ [16]. For the scale-free networks generated by the BA model ($\gamma = 3$), the natural cutoff is $\approx k\sqrt{n}$.

A natural cutoff may not always be achievable due to technical and topological constraints. In practice, there is often a limit (hard cutoff [16]) on the number edges that the nodes can have. Therefore, in this paper we focus on the limited scale-free networks and study the growth models on such networks. This is different than most of the previous works which study the growth of scale-free networks with no hard cutoff.

Moreover, the degree distribution of nodes in the topologies generated by previous growth models complies with only a single exponent γ because the preferential attachment process is designed independent from γ. In this paper, we propose a growth model which defines the connection probability of new joining node to existing nodes according to the desired γ value that will comply with the degree distribution of nodes in the final topology.

In the construction of an overlay topology, it is also important to do the construction efficiently. Even though the growth of scale-free topologies has been extensively studied, less focus is given to the applicability and construction overhead of growth models. In a real network application (such as peer-to-peer networks), the growth of such scale-free overlay topologies may cause high communication overhead between nodes. Whenever a new node joins to the network, it needs the current degree information of all nodes (global topology information) to compute $p_c(j)$ for each existing node j and to select which nodes it will connect. Different than this working principle, in [3], Bent et al. propose a new growth model which can reduce the communication cost. When a new node joins the network, it sends a network-wide broadcast message (using flooding) to announce its presence. Then, each existing node receiving this broadcast message sends a response message after a response time[3] that is inversely proportional to the node's degree, d_i, expires. Finally, once the new node starts to receive the responses from existing nodes, it connects to the first k responders (since each new node connects only k of the existing nodes at its joining time). This type of preferential attachment method is a special case of computing by time [19] that has an extra benefit of reducing the messaging traffic during new node connection process. Each existing node that has already sent or forwarded k different responses (of other nodes or its own) to the newly joining node can stop forwarding any other responses, since they will not have any chance to be selected for connection.

Another type of algorithms which aim to reduce the construction overhead adopt the idea of locality. In LLR algorithm [18], authors propose to use the BA growth model with the following modification. Only first $x\%$ of all nodes when ordered according to their hop distances are used to compute the connection probability ($p_c(j)$) of each node. Therefore,

[1]The diameters of limited scale-free networks are naturally bigger and scale differently than the diameters of scale-free networks without hard cutoffs.

[2]The idea of preferential attachment indeed is equivalent to Yule process [14], which is used to model the distribution of sizes of biological taxa. Its first application to growth of networks (specifically to citation networks) is by Price [13] under a mechanism called 'cumulative advantage'. The name 'preferential attachment' and its popularity as scale-free network models is because of Barabasi and Albert's work [12] which indeed independently rediscovered the same growth model on the web.

[3]The response time, t_d, is uniformly distributed between 0 and an upper value, $t_u = \frac{t_0}{d_i}$, where t_0 is a constant.

only the nodes in the vicinity of new node try to connect to it. Even though this algorithm helps in decreasing the construction overhead, it causes divergence from scale-free network topology. This is because some high degree nodes, which indeed need to have high connection probability to new joining nodes according to preferential attachment idea, may not be close to the new node, thus they might be out of that predefined $x\%$. Similarly in [16], Guclu et al. propose algorithms for building scale-free overlay structures for peer-to-peer networks considering the locality in the preferential edge assignment. For example, in the Hop-and-Attempt preferential attachment (HAPA) algorithm they propose, each new joining node first selects a random node and attempts to connect it. If it can not achieve connection (due to hard cutoff and preferential selection probability) or it needs more nodes to connect (to fill all its k stubs), it jumps to a random neighbor of previous node and attempts to connect to it. This continues until the new node fills all its stubs. Even tough this algorithm works locally, it still assumes that the nodes know the total node count (n) in the network. The new node selects a random number between 0 and 1 and it decides to connect to a visited node j if that random number is less than $p_c(j) = \frac{d_j}{\sum_{i=1}^{n} d_i}$, where the denominator is indeed equal to $2nk$ (as a new node joins the network, $2k$ edges are added to the total edge (degree) count of the network). Moreover, since $p_c(j)$s gets smaller as n increases, as we will show in simulation section, this algorithm may cause a new node's connection attempt message to visit other nodes in the network several times until it fills all its stubs. As a result, it may sometimes incur higher cost than the cost of a network-wide broadcast message.

Unlike in the previous work, in our approach, the new node first decides the node degrees that each of its k edges should connect to and then selects the nodes among those that have the desired degrees. Here, note that among all nodes with a specific candidate degree, selecting the one with closest hop distance (which will be the most probable case considering the arrival and reply of such connection message) from the new joining node provides extra benefit of alignment with underlying physical layer but does not disturb the scale-free property of the topology.

III. Analysis

In this section, we analyze the growth of limited scale-free networks with n nodes, minimum degree of each node k and exponent of power law γ. By definition, in such networks, the nodes with maximum degree (hard cutoff is denoted by m) form a separate group from other nodes in terms of degree distribution:

$$\begin{aligned} P(i) &= ci^{-\gamma} \text{ for nodes with degree } k \leq i < m \\ P(i) &= 1 - \sum_{j=k}^{m-1} P(j) \text{ for } i = m \end{aligned}$$

where c is a constant. Note that, in this definition, only the nodes that have not yet reached degree m are guaranteed to comply with the power-law degree distribution.

Our goal is to construct a topology that shows perfect adherence to scale-free property. Moreover, we want to achieve this without using any global information. We characterize such a graph by its parameters: n, m, k and γ, defined above. It is easy to show that the following inequalities must hold: $m > 2k$ (we excluded here the trivial case of $m = 2k$ in which all nodes of the graph are of degree m, trivially satisfying the definition of power law distribution of node degrees), $\gamma > 0$, and $n > k$. We are interested in generated graphs with the number of nodes in the range $k < n \leq n_{max}$ and we assume that $n_{max} >> k$.

In this paper, we assume a constant integer k for the number of edges added by each joining node. However, it is a matter of simple extension to have instead a vector $[k_i]$ of expected frequencies with which i edges are added with the newly added node such that $k = \sum_{i=1}^{m} ik_i$.

The three constants, k, m, γ are independent of each other except that for certain values of m, and k, there is a lower bound for γ's.

Let n_i denote the number of nodes with degree i in the network with n nodes. By enumeration of all nodes:

$$n = \sum_{i=k}^{m} n_i \tag{1}$$

By enumeration of all edges (each edge belongs to two nodes):

$$2kn = \sum_{i=k}^{m} in_i \tag{2}$$

Substituting n in Eq. 2 with Eq. 1, and taking n_m out, we get:

$$n_m = \frac{1}{m-2k} \sum_{i=k}^{m-1} (2k-i)n_i \tag{3}$$

The power law degree distribution also yields the equation:

$$n_i = \frac{cn}{i^\gamma} \text{ for } i < m \tag{4}$$

Using Eq. 4 to substitute n_i in Eq. 3, we get:

$$n_m = \frac{cn}{m-2k} \sum_{i=k}^{m-1} \frac{2k-i}{i^\gamma} \tag{5}$$

Using enumeration of nodes ($n_m + \sum_{i=k}^{m-1} n_i = n$) with different node degrees, we can compute the constant c as:

$$c = \frac{m-2k}{\sum_{i=k}^{m-1} \frac{m-i}{i^\gamma}} \tag{6}$$

Note that in limited scale-free networks we cannot enforce the power-law distribution for the nodes with maximum degree m because their frequency is defined by Eq. 5. However, for a given m and k, nodes with maximum degree will also have frequency defined by the power-law ($n_m = cn/m^\gamma$) if γ satisfies:

$$\frac{m-2k}{m^\gamma} = \sum_{i=k}^{m-1} \frac{2k-i}{i^\gamma} \tag{7}$$

Since $m > 2k$, the left hand side of Eq. 7 is always positive, its derivative for γ is $-\ln(m)(m-2k)/m^{\gamma}$ while its value approaches $(1-2k/m)m^{-\gamma+1}$ when γ tends to infinity. The right hand side of this inequality can be initially negative, but for large γ it must be positive. Its value approaches $k^{-\gamma+1}$ when γ tends to infinity and it has the derivative $-\sum_{i=k}^{m-1}\ln(i)\frac{2k-i}{i^{\gamma}}$. It is easy to show that the right hand side decreases slower than the left hand side and therefore at most one unique value of γ can satisfy Eq. 7. The unique solution exists if and only if for γ=0, the right hand side is smaller than the left hand side, $m - 2k \geq 2k(m-k) - (m-1)m/2 + k(k-1)/2$ which reduces to $(m - 2k + 1/2)^2 \geq k^2 + k - 1/2$ and since $k^2 < k^2 + k - 1/4 < (k + 1/2)^2$ then we get $m \geq 3k$. Thus, only for m greater or equal to $3k$, there exists a unique value of γ for which the constructed graph will have power-law distribution of all node degrees (including the nodes with maximum degree m).

Now, we will work on the general case where the frequency of nodes with maximum degree does not need to comply the power-law distribution. To be independent of the graph size n, we will use frequency $f_i = n_i/n$ of nodes with degree i. Then, substituting c in n_i definition with Eq. 6, we get:

$$f_i = \frac{m-2k}{i^{\gamma}\sum_{j=k}^{m-1}\frac{m-j}{j^{\gamma}}} \text{ for } i < m \tag{8}$$

and

$$f_m = 1 - \sum_{i=k}^{m-1} f_i \tag{9}$$

Eq. 8 and Eq. 9 express frequencies, f_i's, as simple functions of m, k and γ.

Let's consider now a growth of the graph from its size of n nodes to the size of $n+1$ nodes. The added node has k edges originating from it which are then connected to the existing nodes, so on average it increases by 1 the number of nodes with degree k, i.e. n'_k=$n_k + 1$.

Let a_i denote the average number of nodes that increase their degree from i to $i+1$ in one step of growth (so the number of nodes of degree i decreases by a_i while the number of nodes with degree $i+1$ increases by a_i) by connecting to a newly added node. Of course, each existing node can add at most one connection to a newly added node. Hence, we have $f_k(n+1)$=$f_k n + 1 - a_k$, so $f_k = 1 - a_k$.

Similarly, f_i = $a_{i-1} - a_i$ for $k < i < m-1$, so by induction:

$$a_i = 1 - \sum_{j=k}^{i} f_j \text{ for } k \leq i < m-1 \tag{10}$$

Finally, a_{m-1} = f_m.

All frequencies must be positive. For that to hold[4], it is necessary and sufficient that $\sum_{i=k}^{m-1}\frac{2k-i}{i^{\gamma}} \geq 0$, which can be rewritten as:

$$2k\sum_{i=k}^{m-1} i^{-\gamma} \geq \sum_{i=k}^{m-1} i^{-\gamma+1} \tag{11}$$

[4]Extended details and proofs are presented in our technical report [17].

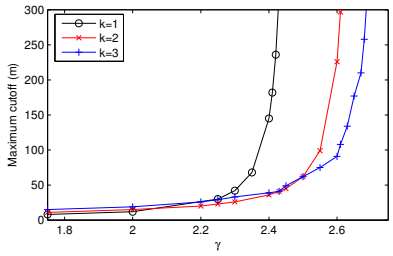


Fig. 1. Maximum cut off (m) values for different k's in perfectly growing scale-free graph.

If this condition is not satisfied, it is always sufficient either to appropriately increase γ or k or to sufficiently decrease m. Other changes to these parameters may or may not, depending on the particular values of the parameters, also cause the inequality of Eq. 11 to be satisfied. It is easy to notice that for $\gamma \geq 3$ this inequality is satisfied for arbitrary m and k. Figure 1 plots the maximum values of m for given γ and k values. It confirms that the maximum value of m goes to infinity for $\gamma \geq 3$.

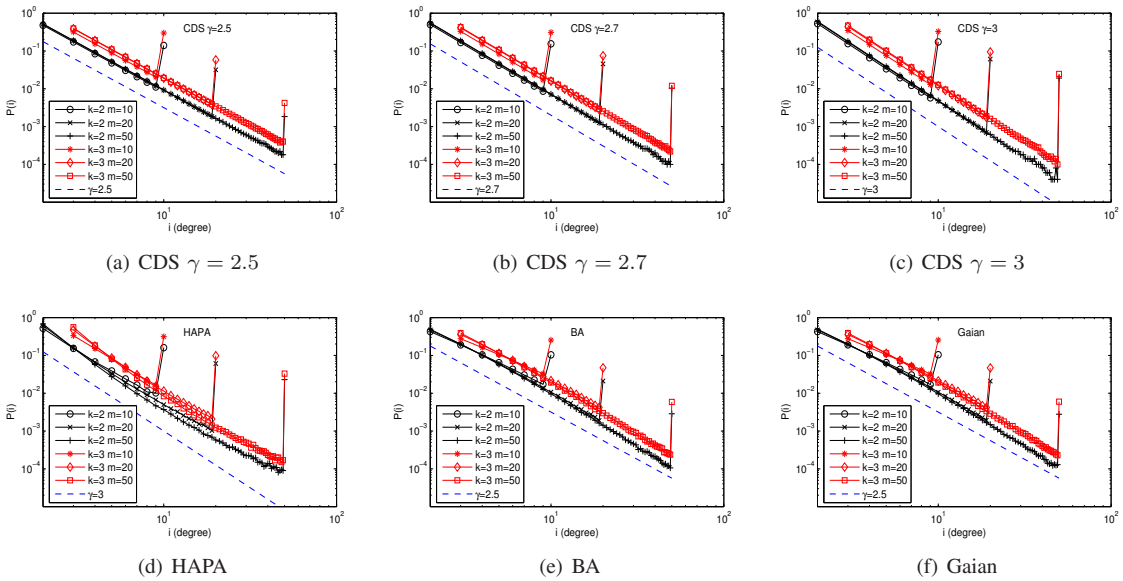
IV. Proposed Growth Algorithm

Following the analysis in previous section, here we present our *Candidate-Degree-Selection (CDS)* based growth algorithm for constructing overlay topologies with power-law degree distribution of desired γ. The algorithm is distributed and uses randomness. It starts with an initial configuration of a fully connected graph of $k+1$ nodes. When a new joins the network, it randomly decides the degrees of nodes that it will connect. To this extend, it generates k random numbers, $r_1 \ldots r_k$, each in range [0,1] and finds the degree that each random number corresponds to. We define v_i's as:

$$\begin{aligned} v_{k-1} &= 0 \\ v_i &= \left(v_{i-1} + \frac{a_i}{k}\right) \ \forall\ k \leq i \leq m-1 \end{aligned}$$

where a_i's are computed in previous section. The random number r_i corresponds to the degree l such that $v_{l-1} \leq r_i < v_l$ is satisfied. Having computed all k node degrees that it wants to connect, the new node then broadcasts a message with these degree values in the message. Once the existing nodes in the network receive such a message, the nodes of the desired degrees respond to the new node to make a connection with the new node. Then the new node selects the first k of the nodes with desired degrees and connects them. Here, note that, there may not exist nodes with the desired degrees, which is likely only at the earlier stages of the network growth, until the first node reaches the degree m. In such cases, new node broadcasts a special request for the lower degree nodes[5], after the period of response for the original broadcast passes.

[5]Such a broadcast will be run only a limited number of times over the initially small network, so its impact on the communication overhead is negligible.



(a) CDS $\gamma = 2.5$ (b) CDS $\gamma = 2.7$ (c) CDS $\gamma = 3$

(d) HAPA (e) BA (f) Gaian

Fig. 2. Degree distributions in different growth models ($n = 5\text{x}10^4$).

V. Simulations

To compare the proposed growth model with existing algorithms, we generated different topologies (consisting of n nodes) using different k, m and γ values. We start with a fully connected network of $k+1$ nodes and add a new node to the network following the connection mechanisms of each growth model. The new node selects k of the existing nodes (which have not reached its maximum edge limit) according to the algorithm in use and connects to them.

The algorithms we compare in the simulations are listed in Table I. As the table shows, BA model needs the global topology information (degrees of all nodes). Even though HAPA model does not need degrees of each node, it still needs the global knowledge of total node or edge count in the network. On the other hand, Gaian and CDS models do not use any global knowledge. While other models can only generate a network of fixed degree distribution exponent (γ), CDS model can create topologies with desired exponent (so with desired network properties such as diameter).

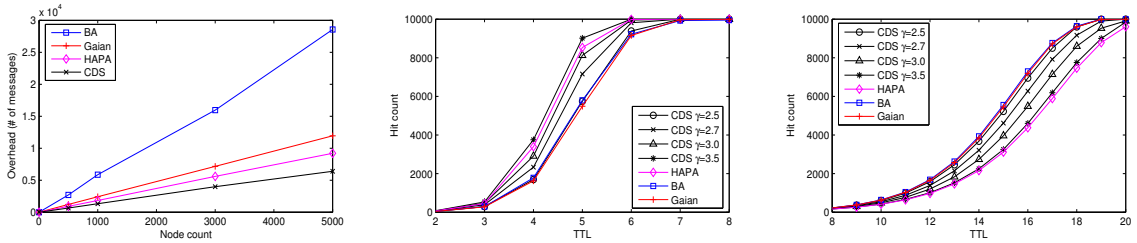
Model	Global knowledge used	Flexible exponent (γ)
BA [12]	Degrees of all nodes	No
HAPA [16]	Total node count	No
Gaian [3]	None	No
CDS	None	Yes

TABLE I
Comparison of Growth Models

In Figure 2, we show the degree distribution in topologies constructed by the compared algorithms. Since our algorithm can produce a scale-free network with a desired γ exponent, we generated several network topologies with different γ values. However, the other algorithms can yield a network only with a single γ value. The figure clearly shows that the topologies created by our algorithm perfectly match with the desired degree distribution of used γ values in the construction. On the other hand, the other algorithms can not achieve a good scale-free distribution even though some use global topology information during construction. There is a curve rather than a line in their results.

In Figure 3(a), we present the communication overhead (e.g. number of messages) during the construction of a scale-free network by each algorithm. In all algorithms except HAPA, when a node wants to join the network, it sends a broadcast message to announce its presence. Then, in BA algorithm every node sends its current degree count back to the new joining node. In Gaian algorithm, as we mentioned before, each node only sends (or forwards) at most k messages (containing degree of the corresponding node) towards the new joining node. This is also true in our algorithm but in our algorithm only the nodes with desired degree respond, so the communication overhead is lower than in the Gaian model. In HAPA algorithm, the new joining node first selects a random node and attempts to connect. Then it randomly walks in the network through neighbors until all its stubs are filled (i.e. k eligible nodes are found to connect). Even though HAPA algorithm is a localized algorithm, since each connection attempt by new node becomes successful by preferential attachment rule (i.e. with probability $p_c(j)$) and only if the attempted node has edge count lower than the hard cutoff value, the new node's connection attempt message needs to travel a lot (sometimes a node is visited several times). Thus it results in a large messaging overhead[6]. Figure 3(a) shows that among all algorithms, the overhead in our algorithm is the

[6]The overhead of HAPA algorithm in Figure 3(a) does not include the overhead that will be generated for maintenance of total node count information at each node of the network. Its overhead will be much higher if that would also be included.



(a) Communication overhead ($k = 2$, m=10)

(b) FL search ($k = 2$, m=50)

(c) NF search ($k = 2$, m=50)

Fig. 3. Comparison of all algorithms according to different metrics.

smallest. When we consider this result with the perfect degree distribution our algorithm achieves with a given exponent and without any global information, we can clearly state its superiority over other algorithms.

Finally, we compare the algorithms in terms of search efficiency in unstructured peer-to-peer networks. We utilized flooding (FL) and normalized flooding (NF) type searching to find average hit ratios on the networks constructed by the compared algorithms. In FL search the source node sends a message to all its neighbors. If the neighbors do not have the requested item, they send the message to their neighbors, excluding the source node. This process is repeated a certain number of times (TTL). However, in NF search [20], when a node receives the message, it forwards the message only to randomly chosen k neighbors, except the one that forwarded the message. Thanks to this limited forwarding, the messaging redundancy caused by FL search is reduced while remarkably good hit ratios are achieved. When we compare the FL and NF based search efficiency in the topologies (with $n = 10k$ nodes) constructed by the compared algorithms, we observe the following results. While in FL based search (Figure 3(b)), our algorithm with $\gamma = 3.5$ (HAPA algorithm is closest to it) achieves the best hit ratios, in NF based search (Figure 3(c)), our algorithm with $\gamma = 2.5$ together with BA and Gaian algorithms achieves the best hit ratios. Since our algorithm can create a perfect scale-free network with a desired γ value, the γ value that gives the best search efficiency for the given search algorithm can be used to create the scale-free overlay topology and the performance of searching can be increased.

VI. Conclusion

In this paper, we introduced a new algorithm for growing limited scale-free overlay network topologies. Unlike previous growth algorithms, it can create a topology with a given γ value and the topology created shows perfect adherence to scale-free degree distribution. It does not use any global information and its messaging overhead is lower than previous algorithms. It can also provide better search efficiency once its γ value is adjusted according to the search algorithm. In future work, we plan to look at the network characteristics when node joins and leaves occur at the same time. Moreover, we will also show the benefit of our algorithm's flexible-γ-based topology creation ability on other applications.

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